Aalto University

Mathematics of Juggling / Spring 2015

Exercises for Thu Apr 23

Some of the problems will become clear during the lectures (Tue 21 - Wed 22), and others may be postponed until next week. The role of the exercise sessions is to complement the lectures and discuss about the projects.

Problem 1: Let $a_0, \ldots, a_{n-1}$ be a siteswap, and define a corresponding affine permutation $f : \mathbb{Z} \to \mathbb{Z}$ by

$$f(t) = a_{t \bmod n} + t - b,$$

where $b = \frac{1}{n} \sum_{i=0}^{n-1} a_i$. Prove that the set of affine permutations constructed from n-periodic siteswaps in this way form a group under function composition.

Problem 2: Prove that the order of the affine Weyl group $\widetilde{A}_{n-1}$ is infinite for any n.

Problem 3: How many shift cycles are there in a) the $(4, 7)$ state graph, b) the $(4, 8)$ state graph? What are the lengths of the shift cycles?

Problem 4: Let $n = n(b, h)$ be the number of shift cycles in the (b, h) state graph. Prove that a prime pattern cannot be longer than $\binom{h}{b} - n$.

Problem 5: Prove that a strongly connected state graph is aperiodic if it contains a simple loop (= an arrow from a state to itself).

Problem 6: Prove that the product of two Markov matrices is a Markov matrix.