



Aalto University

Mathematics of Juggling / Spring 2015

Exercises for Thu Apr 30

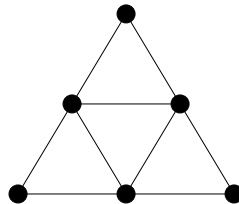
Problem 1 was left partially open and Problem 2 was skipped last week. Problems 3 and 4 relate to the lecture on Tue Apr 28. Problems 5 and 6 relate to the lecture on Wed Apr 29.

Problem 1: Let $n = n(b, h)$ be the number of shift cycles in the (b, h) state graph. Prove that if $b \geq 2$, a prime pattern cannot be longer than $\binom{h}{b} - n$.

Problem 2: Prove that a) a probability vector multiplied by a Markov matrix is a probability vector, b) the product of two Markov matrices is a Markov matrix.

Problem 3: Prove that in a finite and irreducible state graph all states have the same period.

Problem 4: A bug walks uniformly randomly in a triangular lattice depicted below. Construct the Markov matrix of the process and determine the long-term visiting frequencies of the lattice points.



Problem 5: Prove that the uniform distribution is stationary for a Markov process with a doubly stochastic matrix.

Problem 6: Prove that the number of ways to extend a state $B \in G(b, h)$ is $\prod_{x \in B} (1 + v_B(x))$.

Problem 7: Let $S(n, k)$ denote the Stirling numbers of the second kind. Prove that the number of extended (b, h) states is $S(h + 1, h - b + 1)$