



Mathematics of Juggling / Spring 2015

Exercises for Thu May 7

Aalto University

No session next week on Thu May 14 (holiday), so this is the last set.

Problem 1: Let $E(b, h)$ satisfy the recursion

$$E(b, h + 1) = E(b, h) + (h - b + 2)E(b - 1, h).$$

Show that $E(b, h) = S(h + 1, h - b + 1)$.

Problem 2: Prove (carefully) that the extended process is irreducible and aperiodic.

Problem 3: Why are there only countably many states in $G(b, \infty)$?

Problem 4: Let $0 < a < 1$. Find the normalizing constant C such that $\mu(\{i, j\}) = Ca^{i+j}$ defines a probability measure on two-element subsets of \mathbb{Z}_+ .

Problem 5: Solve the general stationary equation (i.e. give μ in terms of h)

$$\mu(x) = \mu(x + 1) + \mu(0)h(x)$$

for the unbounded one-particle case.